



**MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1**

Wednesday 5 May 2010 (afternoon)

1 hour 30 minutes

Candidate session number

| | | | | | | | | |
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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. A shipping container is a cuboid with dimensions 16 m, $1\frac{3}{4}$ m and $2\frac{2}{3}$ m.

(a) Calculate the **exact** volume of the container. Give your answer as a fraction. [3 marks]

Jim estimates the dimensions of the container as 15 m, 2 m and 3 m and uses these to estimate the volume of the container.

(b) Calculate the percentage error in Jim's estimated volume of the container. [3 marks]

Working:

Answers:

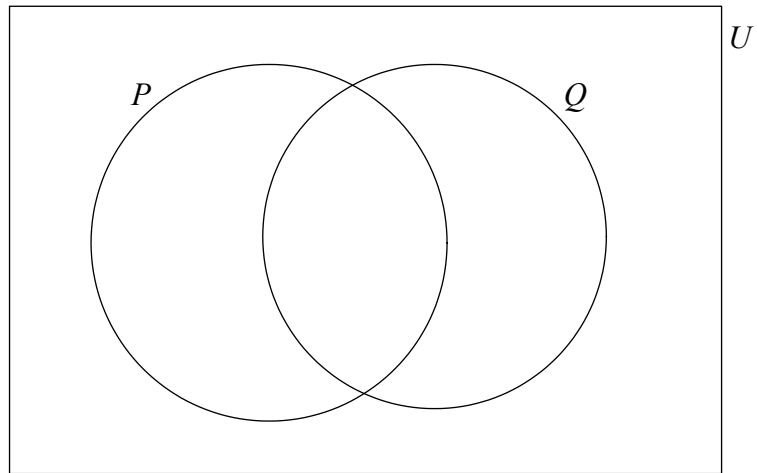
(a) _____

(b) _____



2. The sets P , Q and U are defined as

$U = \{\text{Real Numbers}\}$, $P = \{\text{Positive Numbers}\}$ and $Q = \{\text{Rational Numbers}\}$.



Write down in the correct region on the Venn diagram the numbers

$\frac{22}{7}$, 5×10^{-2} , $\sin(60^\circ)$, 0 , $\sqrt[3]{-8}$, $-\pi$.

[6 marks]

Working:

A large empty rectangular box for working out the placement of numbers in the Venn diagram.



3. Consider two propositions p and q .

(a) Complete the truth table below.

[4 marks]

| p | q | $\neg q$ | $p \Rightarrow \neg q$ | $\neg p$ | $\neg p \Rightarrow q$ |
|-----|-----|----------|------------------------|----------|------------------------|
| T | T | | | | |
| T | F | | | | |
| F | T | | | | |
| F | F | | | | |

(b) Decide whether the compound proposition

$$(p \Rightarrow \neg q) \Leftrightarrow (\neg p \Rightarrow q)$$

is a tautology. State the reason for your decision.

[2 marks]

Working:

Answer:

(b)



4. The stem and leaf diagram below shows the lengths of 22 metal components in cm.

| Stem | Leaf |
|------|---------------------|
| 1 | 2, 2, 3, 7 |
| 2 | 4, 4, 4, 8, 9, 9 |
| 3 | 6, 7, 7 |
| 4 | 1, 1, 1, 1, 3, 5, 6 |
| 5 | 0, 1 |

Key: 1|2 means 1.2 cm

- (a) Write down the modal length of the metal components. *[1 mark]*
- (b) Find the median length of the metal components. *[2 marks]*
- (c) Calculate the interquartile range of the lengths of the metal components. *[3 marks]*

Working:

Answers:

- (a) _____
- (b) _____
- (c) _____



5. The volume of a sphere is $V = \sqrt{\frac{S^3}{36\pi}}$, where S is its surface area.

The surface area of a sphere is 500 cm^2 .

(a) Calculate the volume of the sphere. Give your answer correct to **two decimal places**. [3 marks]

(b) Write down your answer to (a) correct to the nearest integer. [1 mark]

(c) Write down your answer to (b) in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{Z}$. [2 marks]

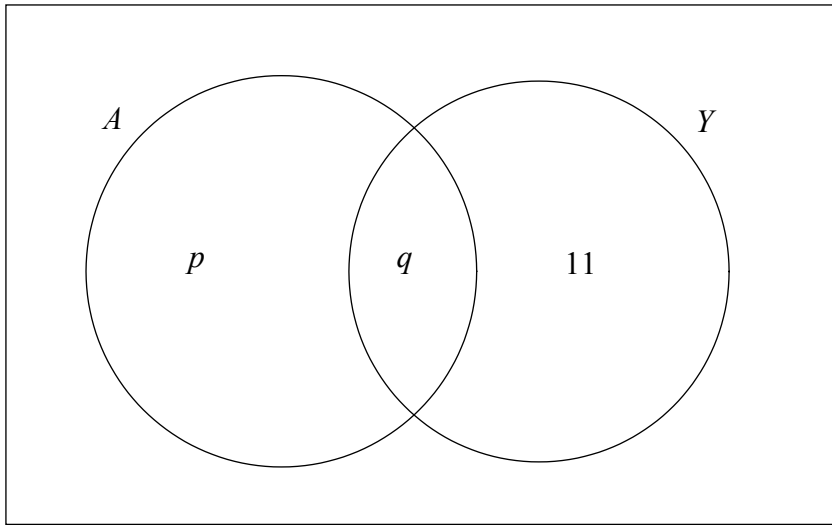
Working:

Answers:

- (a) _____
- (b) _____
- (c) _____



6. A fitness club has 60 members. 35 of the members attend the club's aerobics course (A) and 28 members attend the club's yoga course (Y). 17 members attend both courses. A Venn diagram is used to illustrate this situation.



- (a) Write down the value of q . *[1 mark]*

- (b) Find the value of p . *[2 marks]*

- (c) Calculate the number of members of the fitness club who attend neither the aerobics course (A) nor the yoga course (Y). *[2 marks]*

- (d) Shade, on your Venn diagram, $A' \cap Y$. *[1 mark]*

Working:

Answers:

- (a) _____
- (b) _____
- (c) _____



7. The diagram shows a triangle ABC in which $AC = 17$ cm. M is the midpoint of AC. Triangle ABM is equilateral.

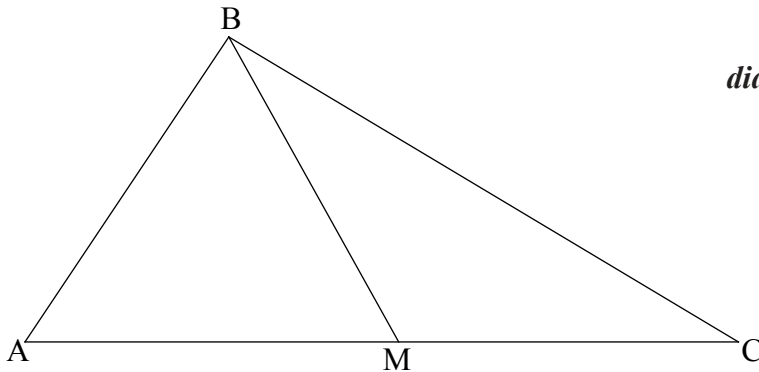


diagram not to scale

- (a) Write down
- (i) the length of BM in cm;
 - (ii) the size of angle BMC;
 - (iii) the size of angle MCB. *[3 marks]*
- (b) Calculate the length of BC in cm. *[3 marks]*

Working:

Answers:

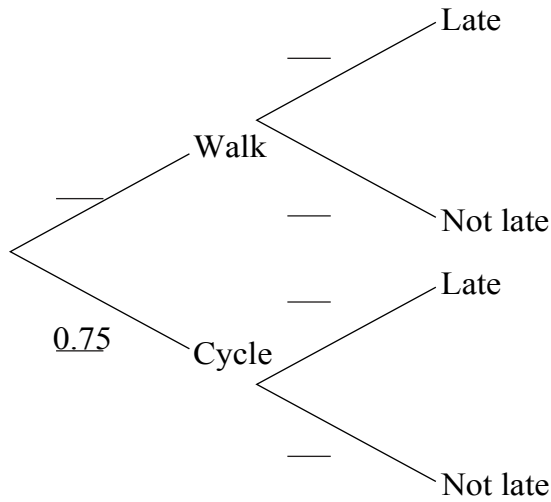
- (a) (i) _____
(ii) _____
(iii) _____
- (b) _____



8. Maria travels to school either by walking or by bicycle. The probability she cycles to school is 0.75.

If she walks, the probability that she is late for school is 0.1.
If she cycles, the probability that she is late for school is 0.05.

(a) Complete the tree diagram below, showing the appropriate probabilities. [3 marks]



(b) Find the probability that Maria is late for school. [3 marks]

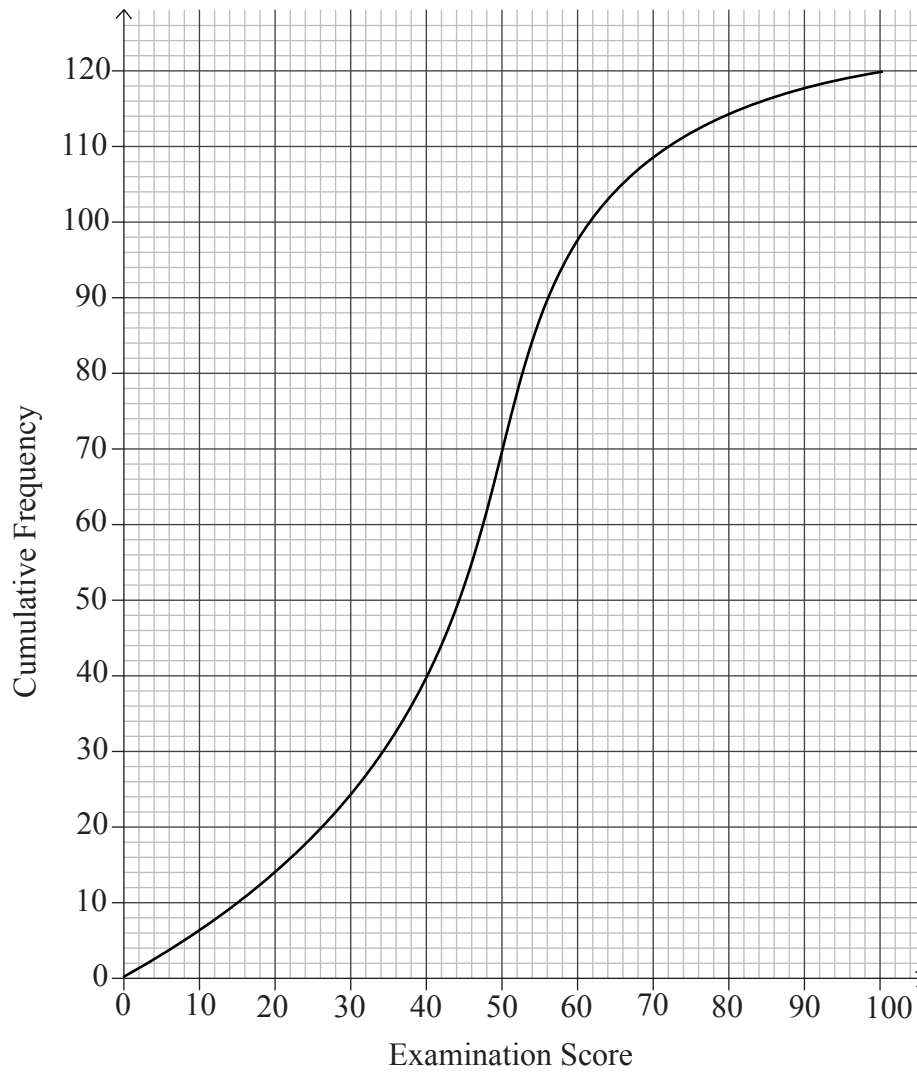
Working:

Answers:

(b) _____



9. 120 Mathematics students in a school sat an examination. Their scores (given as a percentage) were summarized on a cumulative frequency diagram. This diagram is given below.



(This question continues on the following page)



(Question 9 continued)

- (a) Complete the grouped frequency table for the students. [3 marks]

| Examination Score x (%) | $0 \leq x \leq 20$ | $20 < x \leq 40$ | $40 < x \leq 60$ | $60 < x \leq 80$ | $80 < x \leq 100$ |
|---------------------------|--------------------|------------------|------------------|------------------|-------------------|
| Frequency | 14 | 26 | | | |

- (b) Write down the mid-interval value of the $40 < x \leq 60$ interval. [1 mark]
- (c) Calculate an estimate of the mean examination score of the students. [2 marks]

Working:

Answers:

- (b) _____
- (c) _____



10. Tony wants to carry out a χ^2 test to determine whether or not a person's choice of one of the three professions; engineering, medicine or law is influenced by the person's sex (gender).

(a) State the null hypothesis, H_0 , for this test. [1 mark]

(b) Write down the number of degrees of freedom. [1 mark]

Of the 400 people Tony interviewed, 220 were male and 180 were female. 80 of the people had chosen engineering as a profession.

(c) Calculate the expected number of female engineers. [2 marks]

Tony used a 5 % level of significance for his test and obtained a p -value of 0.0634 correct to 3 significant figures.

(d) State Tony's conclusion to the test. Give a reason for this conclusion. [2 marks]

Working:

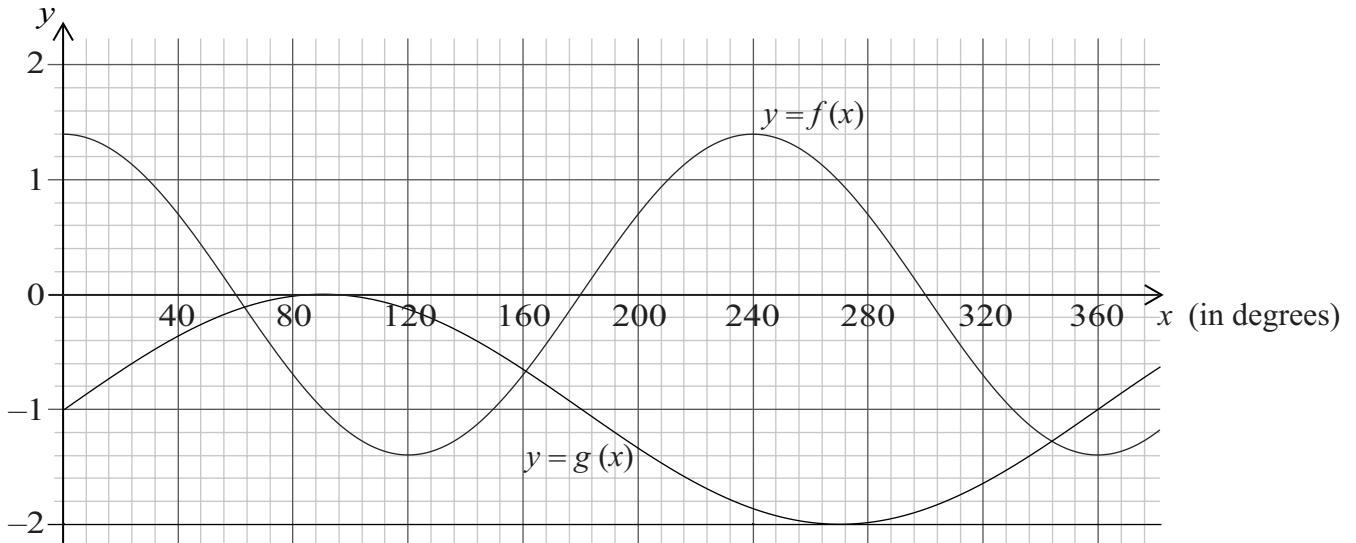
Answers:

- (a) _____
- (b) _____
- (c) _____
- (d) _____



11. The diagram below shows the graph of the functions:

$$f(x) = p \cos(qx), \text{ where } p, q \in \mathbb{Q} \text{ and } g(x) = \sin(x) - 1.$$



- (a) Write down the period of $f(x)$. [1 mark]
- (b) Write down the value of p . [1 mark]
- (c) Calculate the value of q . [2 marks]
- (d) Use your graphic display calculator to find any solutions to the equation $f(x) = g(x)$ in the interval $180^\circ \leq x \leq 360^\circ$. [2 marks]

Working:

Answers:

- (a) _____
- (b) _____
- (c) _____
- (d) _____



12. Susi travels from Singapore to Thailand and changes 1500 Singapore dollars (SGD) to Thai baht (THB). The exchange rate is 1 SGD buys 21.03464 THB.

- (a) Calculate the number of Thai baht Susi buys. Give your answer **correct to the nearest baht**.

[2 marks]

Susi leaves Thailand and travels to Indonesia. She has 20 000 THB and uses these to buy Indonesian rupiah (IDR). The exchange rate is 3.28352 THB buys 1000 IDR.

- (b) Calculate the **total** number of Indonesian rupiah Susi receives, **correct to the nearest thousand rupiah**.

[2 marks]

Susi wants to find the approximate exchange rate between Singapore dollars and Indonesian rupiah and uses the exchange rates for Thai baht to do this.

- (c) Calculate Susi's exchange rate between Singapore dollars and Indonesian rupiah. Give your answer in the form 1 SGD buys x IDR, where x is given correct to the nearest rupiah.

[2 marks]

Working:

Answers:

(a)

(b)

(c)



13. In a research project on the relation between the gender of 150 science students at college and their degree subject, the following set of data is collected.

| | | Degree Subject | | |
|--------|--------|----------------|---------|-----------|
| | | Biology | Physics | Chemistry |
| Gender | Male | 40 | 16 | 35 |
| | Female | 15 | 24 | 20 |

Find the probability that a student chosen at random

- (a) is male; [2 marks]
- (b) is either male or studies Chemistry; [2 marks]
- (c) studies Physics, given that the student is male. [2 marks]

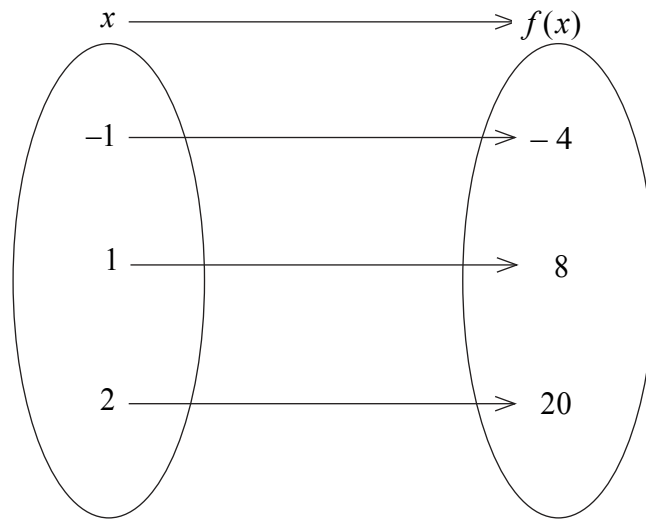
Working:

Answers:

- (a) _____
- (b) _____
- (c) _____



14. A quadratic function, $f(x) = ax^2 + bx$, is represented by the mapping diagram below.



- (a) Use the mapping diagram to write down **two** equations in terms of a and b . [2 marks]
- (b) Find the value of
 - (i) a ;
 - (ii) b . [2 marks]
- (c) Calculate the x -coordinate of the vertex of the graph of $f(x)$. [2 marks]

Working:

Answers:

- (a) _____

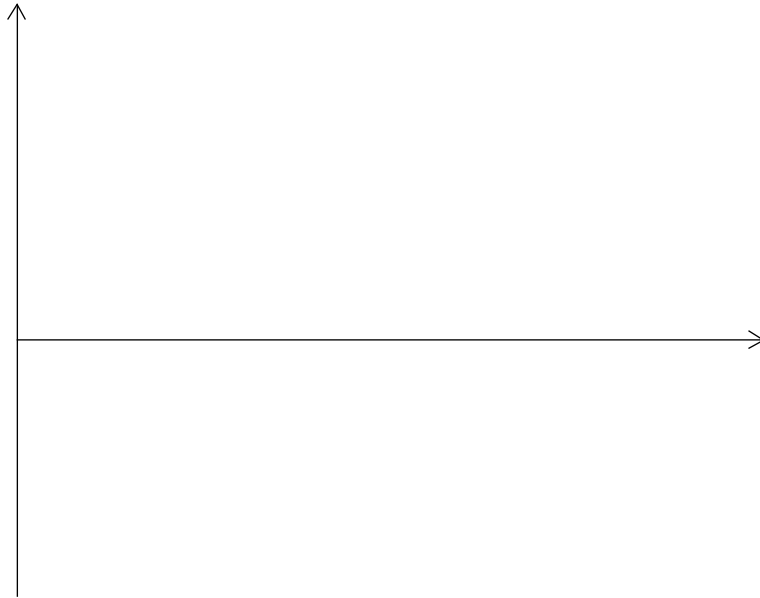
- (b) (i) _____
(ii) _____
- (c) _____



15. The function $f(x) = 5 - 3(2^{-x})$ is defined for $x \geq 0$.

(a) (i) On the axes below sketch the graph of $f(x)$ and show the behaviour of the curve as x increases.

(ii) Write down the coordinates of any intercepts with the axes. [4 marks]



(b) Draw the line $y = 5$ on your sketch. [1 mark]

(c) Write down the number of solutions to the equation $f(x) = 5$. [1 mark]

Working:

Answers:

(a) (ii) _____
(c) _____

